This is the three questions for a “take-home” section of the final. UNC Asheville requires all students to be assessed in their writing skills within their major and CSCI 434 is a course where this assessment is required. (I learned this last week.)

Here are **three** problems to solve with nicely written answers. Give a solution for the problem appropriate for your fellow classmates *and* your professor.

All problems are based on a variation of the middle thirds problem (Problem 4.48) of the textbook which involves two languages from the alphabet $\Sigma = \{0,1\}$

- D_1, the "language of all strings that contain a 1 in their middle third"
- D_2, the "language of all strings that contain two 1's in their middle third"

I am re-interpreting this as meaning exactly one 1 in D_1 and two 1's in D_2. I think this makes it a little easier to understand, solve, and illustrate. That is:

- $D_1 = \{xyz \mid x, y, z \in (0+1)^* \text{ and } |x| = |y| = |z|$

  $\text{where } x \in 0^*, y \in 0^*10^*, \text{ and } z \in 0^*\}$
- $D_2 = \{xyz \mid x, y, z \in (0+1)^* \text{ and } |x| = |y| = |z|$

  $\text{where } x \in 0^*, y \in 0^*10^*10^*, \text{ and } z \in 0^*\}$

Or, equivalently, D_1 and D_2 are of size $3n$ for some integer $n \geq 1$ and are composed only of 0’s except that the **middle** third of D_1 contains exactly one 1 and the **middle** third of D_2 contains exactly two 1’s.

For example, 010 and 000100 are in D_1, and 001100 and 000101000 are in D_1, but ϵ, 0, 1, 000, 0110, and 000111000 are in neither D_1 nor D_2.

Here are the three proofs you are to make. Try to keep each at a couple of paragraphs.

Mini Writing Program 1

Show that D_1 is a context-free language with a well-written argument that the following context-free grammar generates D_1.

$$D \rightarrow 010 \mid 00D0 \mid 0D00$$

I suggest using a proof by induction (pp 22-25 of the textbook). Start by thinking of why the 3 strings in D of size 6 can be safely extended to 5 strings of size 9.

Mini Writing Problem 2

Show that D_2 is **not** a context-free language.

Yes, you ***must*** use the Pumping Lemma (Theorem 2.34, p 125).

Remember that you do ***not*** choose the pumping length, the Pumping Lemma does. You **call** the Pumping Lemma and it **returns** *a* pumping length that you may use to cleverly choose a magic string *s* that can be divided into the *uvwxy*.

You can **not** start with something like:

> Consider the string 000100. Let's assume that *p* is 3.

However this is OK:

> Suppose p is the pumping length. Consider the string 0^{p*434}oats1^p ...

Mini Writing Problem 3

Create a Turing machine to decide D_2.

Do ***not*** draw a state diagram, such as the one seen in Figure 3.10 (p 173).

Use Example 3.11 (p 174) as your model. Make a list of numbered actions. Use English to describe these actions. Use phrase "mark the X" (see the discussion of marking on page 175) or "cross off the Y" or "scan to the next Z". "Move to the 2nd X after the 3rd Y" is also OK as is "If X is marked, goto step 7".

What is allowed and not allowed?

It is OK to discuss the algorithms in a general way. For example, you could gather around a whiteboard and animate the actions of the Turing machine. You could also illustrate the kinds of strings that could generated by the grammar shown for Problem 1.

It's a bit like a Literature assignment: You can't copy the phrases of others. Also, you can't write the solution jointly. Remember, I am required to assess **your** writing.

How to turn it in?

We've had three exams given in 100 minute periods. The final is given in a 150 minute period. The in-class part of the final will be targeted for completion in about 75 to 90 minutes. (You are allowed to stay for the entire 150.)

You could try writing these proofs during class, but I ***strongly*** recommend against that. I suggest you write up these proofs and submit them to the Moodle before Thursday, 10 December. I am OK with the usual formats: shared Google Doc, PDF, OpenOffice, MS Word, LaTeX, ...

Bringing a printed copy to class would also be appreciated.