UNCA CSCI 431
Exam 2 Fall 2019
5 November 2019 – 3:15 pm to 4:55 pm

You may use your notes, printouts, scratch paper, and your textbook. You may not use any calculators, electronic devices, or help from any other source or person.

Anyone needing a break during the exam must leave their exam with the instructor.

This exam must be turned in before 4:55 PM.

Name:________________________________

There are five equally-weighted questions.

Problem 1: Regular and context free languages (20 points)

Fixed point numbers are sequences consisting of digits and a single period to represent a few “real” numbers. The period should **not** be the first or last character of the string. Here are some examples:

- 434.001
- 0.1
- 00.1
- 0.000

(Usually 1. and .1 would be allowed, but we are making the problem easier.)

Use the `grep` *expression* `[0-9]` *in place of* `(0+1+2+3+4+5+6+7+8+9)` *in your answers. It’s a **lot** easier to write.*

Part A: Write a regular expression specifying fixed point numbers.

Part B: Write a context free grammar specifying fixed point numbers.

Part C: Using your grammar, draw a parse tree for the fixed point number 11.05 .

Problem 2: CFG → CNF (20 points)

Convert the following Context Free Grammar to Chomsky Normal Form.

- The alphabet for the language is **Σ** = {**a**, **b**}.
- The start variable for the language is (as usual) **S**.

Here are the rules:

- **S → aTa | b**
- **T → bTb | ϵ**

Avoid creativity and stick with the procedure described in Example 2.10 of the textbook as it was followed in last week's class meetings.

Problem 3: CFG → PDA (20 points)

Generate a PDA that accepts the Context Free Grammar of the previous problem:

- **S → aTa | b**
- **T → bTb | ϵ**

I suggest that you follow the **flower algorithm** presented in Theorem 2.20 of the textbook (which you ***must*** have used for your solution to Problem 2.12).

Your answer should be a **real** PDA. Don't label transition arcs with short cuts such as **ϵ, T → bTb** if you expect full credit.

Pumping Lemma (Theorem 1.70)

If L is a context free language, then there is a number *p* (the pumping length) where, if *s* is any string in L of length at least *p*, then *s* may be divided into five pieces $s = uvxyz$, satisfying the following conditions:

- for each $i \geq 0$, $uv^ixy^iz \in A$
- $|v| > 0$ or $|y| > 0$
- $|vxy| \leq p$

Problem 4: Proving Context (20 points)

Show that the language $\{a^ib^jc^k \mid i \geq j \text{ and } i \geq k\}$ is **not** context free.

Note that aabbcc and aac are **in**, but abbc and cab are **out**.

Problem 5: Proving context (20 points)

Show that the language $\{a^i b^j c^k \mid i \geq j + k\}$ is context free. I strongly suggest you create a context free grammar to solve this problem.

Note that aaabbc and aaaab are **in**, but aaabbcc and abba are **not**.

You must add comments to your answer! I'll need them.