Homework 16

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PROBLEM 0

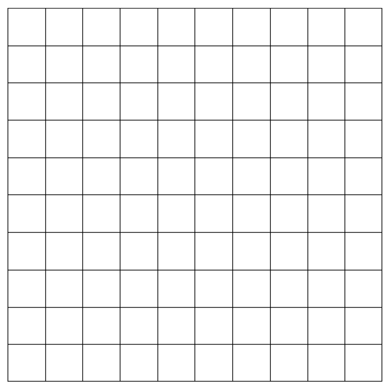

Figure 1: A grid representing N = 10.

PROBLEM 1

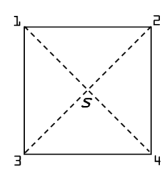

Figure 2: Cell with numbered corners

We may compute the coordinate $< X_i, Y_j >$ for the central point of any cell within the devided unit square by computing the midpoint of eiter of the cell's diagonals. Given the coordinate expressions for the cell's four corner points, we may choose either pair of points that form one of the two existing diagonals. We then use the midpoint formula to solve for the midpoint s as follows. We shall use the expressions of corners 2 and 3 for our calculation.

Corner 2:

$$\left\langle \frac{i}{N}, \frac{(j+1)}{N} \right\rangle \tag{1}$$

Corner 3:

$$\left\langle \frac{(i+1)}{N}, \frac{j}{N} \right\rangle \tag{2}$$

The midpoint formula:

$$\left\langle \frac{X_1 + X_2}{2}, \frac{Y_1 + Y_2}{2} \right\rangle \tag{3}$$

From here it is simply a matter of substitution and simplification.

$$\left\langle \frac{(\frac{i}{N} + \frac{(i+1)}{N})}{2}, \frac{(\frac{(j+1)}{N} + \frac{j}{N})}{2} \right\rangle \tag{4}$$

$$\left\langle \frac{2i+1}{2N}, \frac{2j+1}{2N} \right\rangle \tag{5}$$

The coordinate expression for the central point of a cell within the unit grid is

$$\langle X_i, Y_j \rangle = \left\langle \frac{i + \frac{1}{2}}{N}, \frac{j + \frac{1}{2}}{N} \right\rangle \tag{6}$$

The presence of $\frac{1}{2}$ makes sense intuitively.

PROBLEM 2

Mathematically, we may use the distance formula along with the coordinates for the cell midpoint and origin to determine if the length d of a line segment formed by the two points is less than 1.

$$d = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2} \tag{7}$$

Let $< X_1, Y_1 >$ be the origin, and $< X_2, Y_2 >$ be the cell midpoint. We now have

$$d = \sqrt{\left(\frac{i + \frac{1}{2}}{N}\right)^2 + \left(\frac{j + \frac{1}{2}}{N}\right)^2} \tag{8}$$

PROBLEM 3

We may also solve the previous problem by means of a simplistic test. The Cartesian formula for the unit circle (which happens to be inscribed within a unit square for our case) is

$$1 = X^2 + Y^2 \tag{9}$$

This holds for all X and Y on the circumference of the circle. For those X and Y that lie within the circle, the sum of squares result like above would be *less than 1.* From this, we may form a test expression using the cell midpoint coordinates computed in **Problem 1**.

$$[(X_i * X_i) + (Y_j * Y_j) < 1]$$

PROBLEM 4

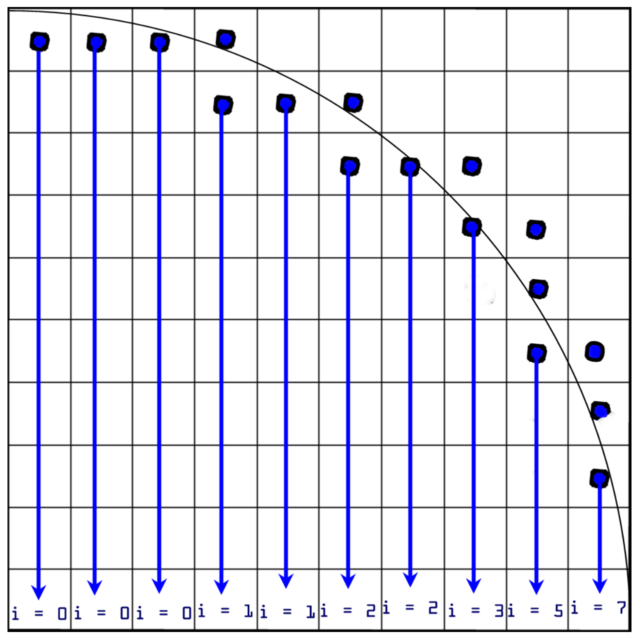

Figure 3: For N = 10; Dotted cells must be tested; Arrows indicate cells counted after midpoint validation; i value after validation used to compute number of counted cells (N-i)

PROBLEM 5

A possible solution in attempted pseudocode :-)

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 This solution uses a columnar method to traverse the tested cells of the
grid.

WHILE (i <=N-1) && (j <=N-1)
====================================
Test if cell midpoint is in unit circle, and act accordingly
IF (midpoint test < 1)
>>>sum of COLUMNAL cells begin w/ CURRENT and go down the row
>>>This would be SUM += (N-i)
>>>increment column counter j
<ITERATE>
ELSE
>>>increment row counter i
<ITERATE>
====================================
RETURN ''sum'' (which, if N → ∞, should equal π/4)
```