Problem 1 (12 points) Adding
Add the following pairs of seven-bit two's complement numbers and indicate which additions result in an overflow.

| 1010111 <br> $+\frac{1110100}{1001011}$ <br> no overflow | $+\frac{1011111}{\substack{01111110 \\ \text { overflow }}}$ |
| ---: | ---: |
| 0011000 <br> $+\frac{0111111}{1010111}$ <br> overflow | $+\frac{1000111}{\substack{0111001 \\ \text { no overflow }}}$ |

Problem 2 (8 points) Two's-complement
Translate the following numbers represented in six-bit two's-complement into their corresponding decimal representation.

$$
\begin{gathered}
100101 \\
-27
\end{gathered}
$$

001100 12

## Problem 3 (12 points) Floating point

Express the following three numbers in IEEE floating point notation. I've left some spaces between the major bit fields in the number. None of these answers should involve long calculations.

$$
\begin{aligned}
& 2.5 \rightarrow 10.1 \rightarrow 1.01 \cdot 2^{1} \\
& 01000000001000000000000000000000 \\
& -2.5 \rightarrow-10.1 \rightarrow-1.01 \cdot 2^{1} \\
& 11000000001000000000000000000000 \\
& 0.25 \rightarrow 0.01 \rightarrow 1.00 \cdot 2^{-2} \\
& 10111110100000000000000000000000
\end{aligned}
$$

## Problem 4 ( 6 points) Ranges

What is the number of different values that can be represented by 7 binary digits?

## 128 or $2^{7}$

What is the largest number that be represented in 7-bit twos-complement notation?

$$
63 \text { or } 2^{6}-1
$$

What is the smallest number that can be represented in 7-bit twos-complement notation?

$$
-64 \text { or }-2^{6}
$$

## Problem 5 (2 points) Extensions

What is the result of extending the following 6-bit two's complement number into an 8-bit two's complement number?

$$
110011 \text {--> } 11110011
$$

Problem 6 ( $\mathbf{3}$ points) Words
Today, the most common use of the word "multiplex" is to describe "a group of two or more motion-picture theaters on the same site or in the same building, esp. a cluster of adjoining theaters" [from dictionary.reference.com]. How is this usage related to the multiplexer we introduced in this course?

In the multiplex theater, several different movie screen share many common facilities, such as movie projection room and concession stands. In the digital multiplexer, many input sources share an output.

## Problem 7 (6 points) Sequential logic elements

The R-S latch has two binary inputs and one binary output. How does the output of the latch change in response to these four different pairs of bits that can be applied to its inputs? Don't describe the "internals". Just describe how the output changes.

First, the two inputs, $R$ and $S$, should never be asserted at the same time. Otherwise, when $R$ is asserted, the output becomes 0 ; and when $S$ is asserted the output becomes 1 . When neither $\mathbf{R}$ nor S is asserted, the output holds its value, which could be either 0 or 1 .

Problem 8 (13 points) Transistors
In the space below, draw a CMOS implementation of a 3-input AND gate.

## later

Problem 9 (3 points) deMorgan's Law
How can deMorgan's Law be used to reduce the number of transistors needed to implement the following very simple circuit with two inputs and one output. [The x 1 is the size, in bits, of each input and output port. You can ignore it for this problem.]

later

Problem 10 ( 13 points) Gates to Truth
Fill in the truth table on the right to represent the gate-level circuit on the left.


| X | Y | Z | Q |
| ---: | ---: | ---: | ---: |
| 0 | 0 | 0 | $\mathbf{1}$ |
| 0 | 0 | 1 | $\mathbf{0}$ |
| 0 | 1 | 0 | $\mathbf{1}$ |
| 0 | 1 | 1 | $\mathbf{1}$ |
| 1 | 0 | 0 | $\mathbf{1}$ |
| 1 | 0 | 1 | $\mathbf{1}$ |
| 1 | 1 | 0 | $\mathbf{1}$ |
| 1 | 1 | 1 | $\mathbf{1}$ |

Problem 11 (9 points) Bitwise operations
Perform the following bit-wise logical operations on 8-bit numbers expressed as two hexadecimal digits. Your answer should also be expressed in hexadecimal.

| $\operatorname{NOT}(C D)$ | $-->$ | 32 |
| :--- | :--- | :--- |
| $\operatorname{AND}(07, B B)$ | $-->$ | 03 |
| OR (07,BB) | $-->$ | BF |

## Problem 12 (13) Truth to Gates

Draw a circuit, at the gate level, that will implement the following truth table, where $\mathrm{A}, \mathrm{B}$, and C are inputs and where Z is the single output.

| A | B | C | Z |
| ---: | ---: | ---: | ---: |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

