

January 17, 2001

## Important announcements

[Class home page](#)

[Homework assignment 1](#)

[Homework assignment 2](#)

## Positional notation in base (or radix) $b$

$d_{n-1} d_{n-2} \dots d_1 d_0$  **is really**  $\sum_{i=0}^{n-1} d_i b^i$   
2001 **is really**  $2*10^3 + 0*10^2 + 0*10^1 + 1*10^0$

## Conversions to/from decimal (base 10)

Random base → base 10

Multiply and add

Base 10 → Random base

Divide and take the remainder

Dealing with base 16 (hexadecimal)

Digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

[\*Practice in a spreadsheet\*](#)

## Bit groupings

In decimal

6122428867 **is written as** 6,122,428,867

Binary to hexadecimal

101101100111011001101100111000011

1,0110,1100,1110,1100,1101,1001,1100,0011

16CECD9C3

## Bases in C and C++

octal	03721
decimal	2001
hexadecimal	0x7D1

## Addition and Subtraction

Addition	Subtraction
$\begin{array}{r} 41573 \\ + 30742 \\ \hline 72315 \end{array}$	$\begin{array}{r} 340010 \\ - 817 \\ \hline 339193 \end{array}$
$\begin{array}{r} 1000101 \\ + 1011101 \\ \hline \end{array}$	$\begin{array}{r} 1000101 \\ - 11001 \\ \hline \end{array}$

*[More practice with a spreadsheet](#)*

# Representing negative numbers

A natural way to represent -17

Subtract 17 from 0 and see what you get!

$$\begin{array}{r} 000000000000000 \\ - 00000000010001 \end{array}$$

Two's complement representation

Binary numbers starting with 1 are *negative*

Expressing a negative number in  $n$ -bit two's complement

Convert to binary	100010
Extend to $n$ bits	000000000100010
Invert all bits	111111111011101
Generate the one's complement	
Add in a one	111111111011101 + 1
Not it's the two's complement	111111111011110

Expressing a positive number in  $n$ -bit two's complement

Convert to binary	100010
Extend to $n$ bits	000000000100010

The first bit is the  $-2^{n-1}$  place

$$d_{n-1} d_{n-2} \dots d_1 d_0 \quad \text{is now really} \quad d_{n-1}(-b^{n-1}) + \sum_{i=0}^{n-2} d_i b^i$$

Other negative number representations

Sign-magnitude

One's complement

# Overflow

8-bit numbers

	unsigned	signed
00000000	0	0
01111111	127	127
10000000	128	-128
11111111	255	-1

Range of 8-bit two's complement is  $-128$  to  $127$   
or  $-2^{8-1}$  to  $2^{8-1}-1$

Range of  $n$ -bit two's complement is  $-128$  to  $127$   
or  $-2^{n-1}$  to  $2^{n-1}-1$

What about

$$\begin{array}{r} 100 + 50 \\ -100 - 50 \\ 100 - 50 \end{array}$$

Detecting overflow

Compare last carry-in with last carry-out